セミナー@RIKEN
7. February 2012

中性子のスピン測定における不確定性
--- ハイゼンベルクの不確定性原理を超えて ---

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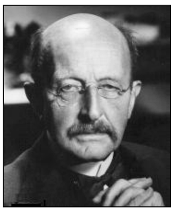
１． イントロダクション
- 量子物理学
- 中性子光学実験

２． 新たな不確定性関係
- ハイゼンベルクの不確定性関係
- 小澤の不等式
- 中性子のスピン測定実験

３． まとめ

Quantum theory



M. Planck

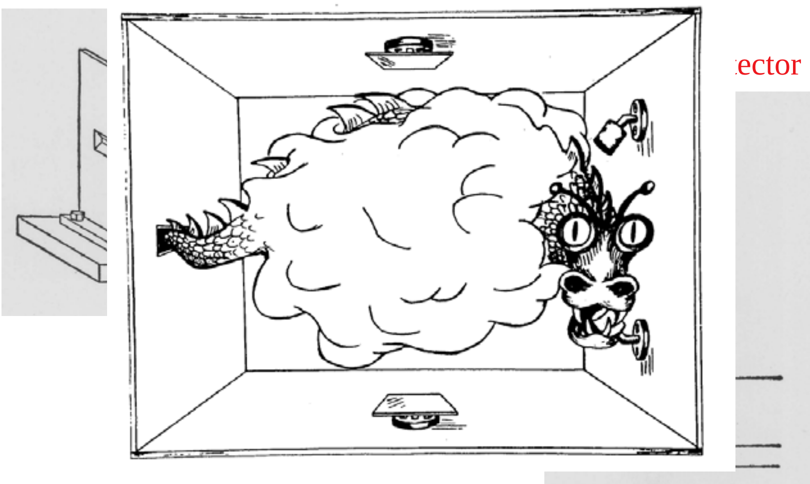
N. Bohr, W. Heisenberg, W. Pauli

L. De Broglie

E. Schrödinger

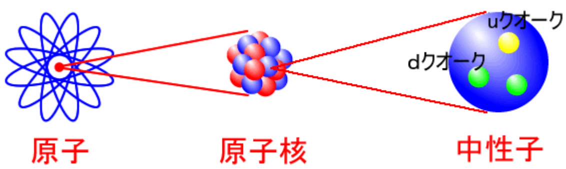
h = 6.6260755(40) 10^{-34} Js (M. Planck)

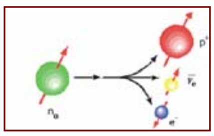
Bohr-Einstein debate(2): which-way detector



Smoky dragon by J.A. Wheeler

中性子：ナノの世界の伝達者





粒子的な性質	波動的な性質
$m = 1.674928(1) \times 10^{-27}$ kg	$\lambda_c = \frac{h}{mc} = 1.319695(20) \times 10^{-15}$ m
$s = \frac{1}{2}\hbar$	
$\mu = -9.6491783(18) \times 10^{-27}$ J/T	$\lambda = 1.8$ Å, $v = 2200$ m/s　（熱中性子）
$\tau = 885.7(0.8)$ s	$\lambda_B = \frac{h}{mv} = 1.8 \times 10^{-10}$ m
u - d - d　クオーク構造	
m 質量,　s スピン, μ 磁気モーメント, τ β-崩壊寿命	λ_c コンプトン波長, λ_B ド・ブロイ波長, v 速度

Neutrons in quantum mechanics

Particle and wave properties

p = mv = h/λ

(L. De Broglie)

Schroedinger equation

$$i\,\frac{\partial\Psi(r,t)}{\partial t} = H\Psi(r,t)$$

(E. Schrödinger)

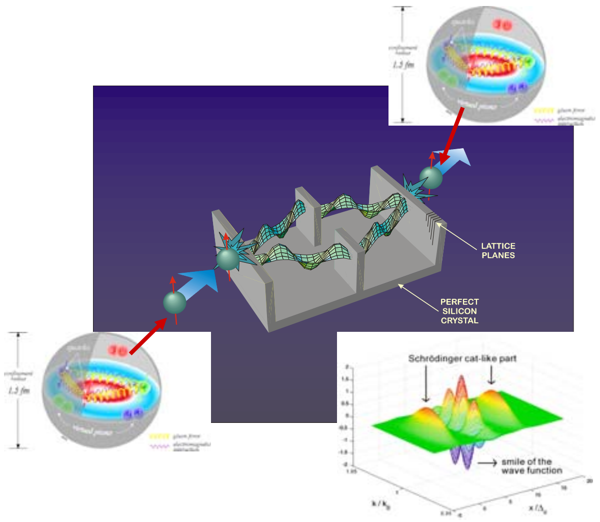
Uncertainity

$\Delta x\,\Delta p \geq h/4\pi$

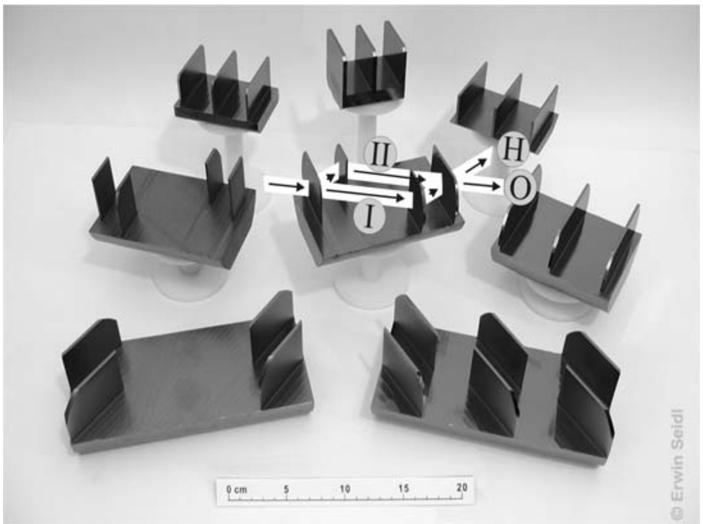
(W. Heisenberg)

M.C.Escher, 1938

Neutron interferometer



Neutron interferometer family



Neutron interferometer experiment(1)

4π-symmetry of spinor wavefunction

Hamiltonian

$\hat{H} = -\mu \cdot \mathbf{B} = -\mu \sigma \cdot \mathbf{B}$

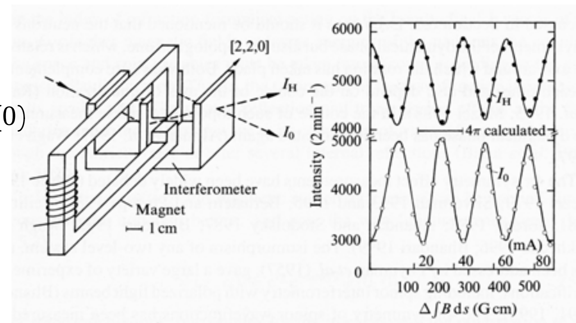
Wave function

$\Psi(t) = \exp[-{}^{iHt}/_{\hbar}] \cdot \Psi(0)$

$= \exp[-{}^{i\mu \cdot \mathbf{B}t}/_{\hbar}] \cdot \Psi(0)$

$= \exp[-{}^{i\sigma \cdot \alpha}/_{2}] \cdot \Psi(0)$

Where $\alpha = \frac{2\mu}{\hbar} \int B\,dt$



H. Rauch et al., PL A**54** (1975) 425.

Neutron interferometer experiment(2)

Gravitationally induced quantum phase

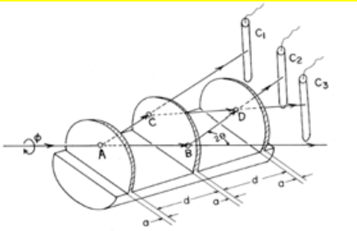
Energy of neutron

gravitational potential

$$E_0 = \frac{\hbar^2 k_0^2}{2m} = \frac{\hbar^2 k^2}{2m} + mgH(\alpha)$$

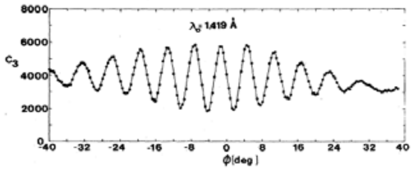
and

$$\Delta k = (k - k_0) \cong -\frac{m^2 gH}{\hbar^2 k_0} \sin\alpha$$



Phase shift

$$\Delta\Phi_{COW} = -2\pi\lambda \frac{m^2}{\hbar^2} g A_0 \sin\alpha$$



R. Colella et al., PRL **34** (1975) 1472; J.L. Staudenmann et al., PR A21 (1980) 1419.

Neutron interferometer experiment (3)

Superposition of spinor wavefunctions

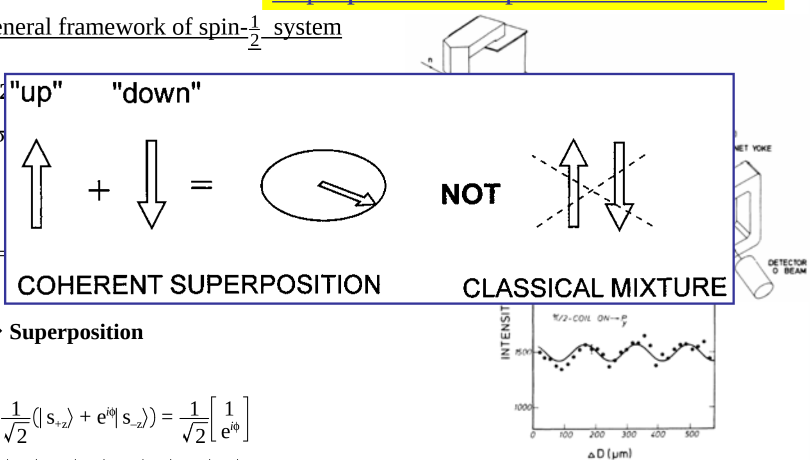
§ General framework of spin-$\frac{1}{2}$ system

(i)

σ

(ii)

$|s_{\pm x}\rangle =$



➔➔ **Superposition**

$$\frac{1}{\sqrt{2}}(|s_{+z}\rangle + e^{i\phi}|s_{-z}\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ e^{i\phi}\end{bmatrix}$$

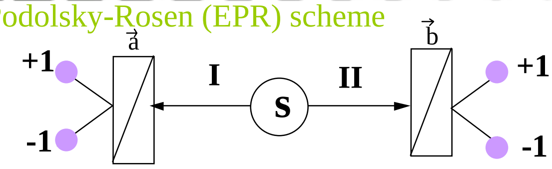
$$|s_{+x}\rangle \rightarrow |s_{+y}\rangle \rightarrow |s_{-x}\rangle \rightarrow |s_{-y}\rangle$$

J. Summhammer et al., PL A**27** (1983) 2532.

Nonlocality in quantum mechanics

Einstein-Podolsky-Rosen (EPR) scheme

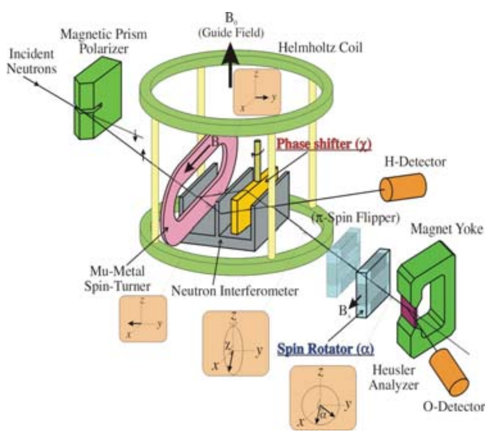
A. Einstein, B. Podolsky and N. Rosen, PR **47**, 777--780 (1935).

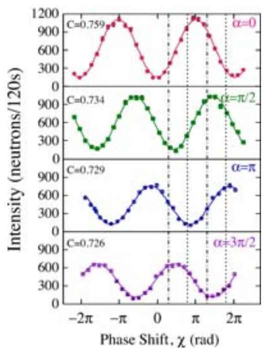


$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_I \otimes |\downarrow\rangle_{II} + |\downarrow\rangle_I \otimes |\uparrow\rangle_{II}\}$$

⇒⇒⇒ Entanglement between *Two-Particles*

Entanglement between two-spaces





$S' \equiv E'(\alpha_1,\chi_1) + E'(\alpha_1,\chi_2) - E'(\alpha_2,\chi_1) + E'(\alpha_2,\chi_2)$

$= 2.051 \pm 0.019 \quad > 2$

Cf. Max. violaion: S'=2.81>2

(newest value: S'=2.202 ± 0.007)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_s \otimes |\,I\rangle_p + |\downarrow\rangle_s \otimes |\,II\rangle_p\}$$

Y. Hasegawa et al., Nature Vol. 425, Sept. 4, 2003

Publications

Nature, Vol. 425, Sept. 4, 2003

letters to nature

Violation of a Bell-like inequality in single-neutron interferometry

Yuji Hasegawa[1], **Rudolf Loidl**[1,2], **Gerald Badurek**[1], **Matthias Baron**[1,2] **& Helmut Rauch**[1]

[1] *Atominstitut der Österreichischen Universitäten, Stadionallee 2, A-1020 Wien, Austria*
[2] *Institute Laue Langevin, B. P. 156, F-38042 Grenoble Cedex 9, France*

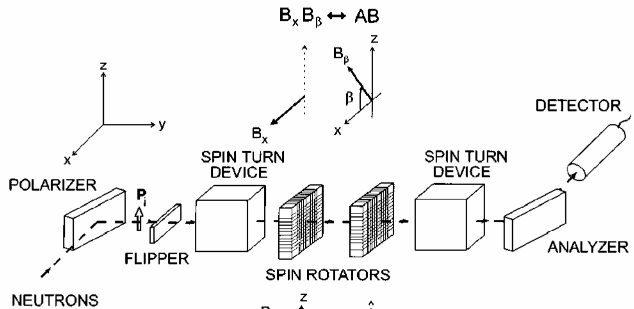
Non-local correlations between spatially separated systems have been extensively discussed in the context of the Einstein, Podolsky and Rosen (EPR) paradox[1] and Bell's inequalities[2]. Many proposals and experiments designed to test hidden variable theories and the violation of Bell's inequalities have been reported[3–7]; usually, these involve correlated photons, although recently an experiment was performed with $^9Be^+$ ions[8]. Nevertheless, it is of considerable interest to show that such

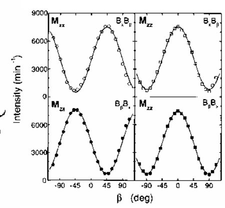
the spinor part[21], that is, different degrees of freedom. The normalized total wavefunction $|\Psi\rangle$, can be represented as a Bell state, $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle \otimes |I\rangle + |\uparrow\rangle \otimes |II\rangle)$. Here, $|\uparrow\rangle$ and $|\downarrow\rangle$ denote the up-spin and down-spin states, and $|I\rangle$ and $|II\rangle$ denote the two beam paths in the interferometer.

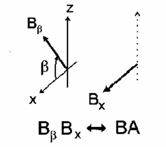
The expectation value for the joint measurement for the spinor $\frac{1}{\sqrt{2}}(|\uparrow\rangle + e^{i\alpha}|\downarrow\rangle)$ and the path $\frac{1}{\sqrt{2}}(|I\rangle + e^{i\chi}|II\rangle)$ is calculated to be:

Frankfurter Allgemeinde Zeitung
10.Sept. 2003

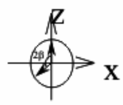
Neutron polarimetry





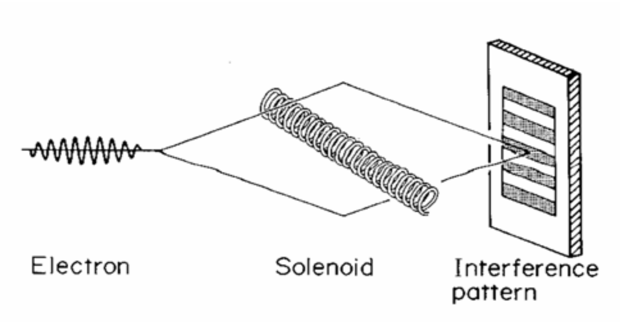


$$\hat{O}|\pm Y\rangle = -e^{\pm i\beta}\cdot|\pm Y\rangle \Rightarrow\Rightarrow$$



$$\hat{O}|+Z\rangle = \hat{O}\left\{\frac{1}{\sqrt{2}}(|+Y\rangle + |-Y\rangle)\right\} = \frac{1}{\sqrt{2}}(e^{i\beta}|+Y\rangle + e^{-i\beta}|-Y\rangle) = \frac{e^{i\beta}}{\sqrt{2}}(|+Y\rangle + e^{-2i\beta}|-Y\rangle)$$

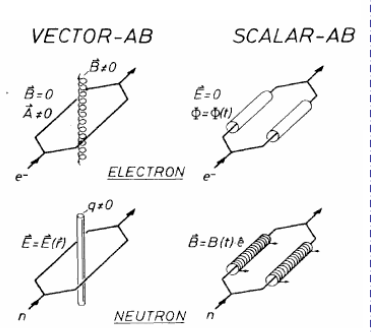
Aharonov-Bohm (AB) effect

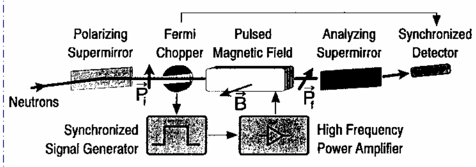


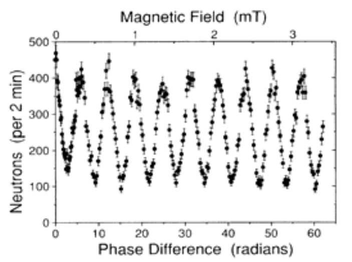
$$\phi_{AB} = \frac{e}{\hbar}\oint \boldsymbol{A}\, \mathrm{ds}$$

Y. Aharonov and D. Bohm,
"Significance of Electromagnetic Potentials in Quantum Mechanics,"
Phys.Rev. **115** (1959) 485-491.

Scalar Aharonov-Bohm (AB) effect

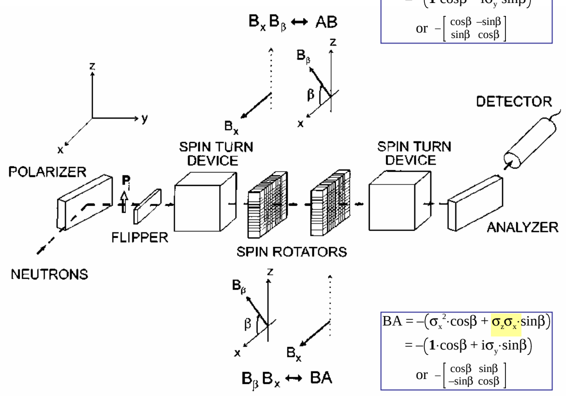






G. Badurek, PRL **71** (1993) 302.

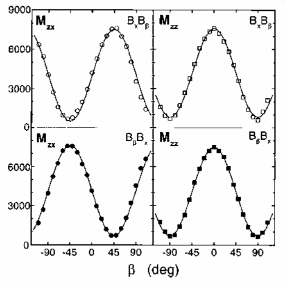
Non-commutation of Pauli spin matrices



$$AB = -(\sigma_x^2 \cdot \cos\beta + \sigma_x\sigma_z \cdot \sin\beta) = -(\mathbf{1} \cdot \cos\beta - i\sigma_y \cdot \sin\beta) \quad \text{or} \quad -\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$

$$BA = -(\sigma_x^2 \cdot \cos\beta + \sigma_z\sigma_x \cdot \sin\beta) = -(\mathbf{1} \cdot \cos\beta + i\sigma_y \cdot \sin\beta) \quad \text{or} \quad -\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$[\sigma_x, \sigma_y] = \sigma_x\sigma_y - \sigma_y\sigma_x = e^{i\frac{\pi}{2}} \cdot \sigma_z - e^{-i\frac{\pi}{2}} \cdot \sigma_z = \left(e^{i\frac{\pi}{2}} - e^{-i\frac{\pi}{2}}\right) \cdot \sigma_z = 2i\sigma_z$$



Y.Hasegawa et al., PLA **234** (1997) 322; PRA **59** (1999) 4614.

Neutrons in quantum mechanics

Particle and wave properties

p = mv = h/λ

(L. De Broglie)

Schroedinger equation

$$i\,\frac{\partial \Psi(r,t)}{\partial t} = H\Psi(r,t)$$

(E. Schrödinger)

Uncertainity

$\Delta x\, \Delta p \geq h/4\pi$

(W. Heisenberg)

M.C.Escher, 1938

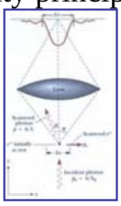
実験の背景

・現代物理学によると、ある２つの物理量を正確に
測定することは原理的に不可能。
＝＞　『不確定性原理』と呼ばれ
　　１００年近く広範に信じられ
　　量子物理学の教科書にも初めに記述されている。

・ {小澤（名古屋大）の理論的予測(2003)
　長谷川(ウィーンエ科大)の中性子光学実験技能
➔２つの最先端研究グループの共同研究の賜物

Uncertainty relation: historical 1

- In 1927 Heisenberg postulated an uncertainty principle:



γ-ray thought experiment

→ $$Q_1 P_1 \approx h$$

→ in modern treatment for commuting observables:

$$\epsilon(Q)\eta(P) \geq \frac{\hbar}{2}$$

ε: error of the first measurmen (Q)

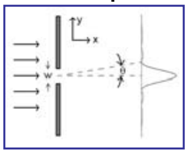
η: disturbance on the second measurement (P)

Uncertainty relation: historical 2

- Kennard considered the spread of a wave function ψ

$$\sigma(Q)\sigma(P) \geq \frac{\hbar}{2}$$

σ: standard deviations



- Robertson generalized the relation to arbitrary pairs of observables in any states ψ

$$\sigma(A)\ \sigma(B) \geq \frac{1}{2}\left|\langle\psi|[A,B]|\psi\rangle\right|$$

→ dependent on the state but independent of the appartus

Is $\varepsilon(A)\eta(B) \geq \frac{1}{2}\left|\langle\psi|[A,B]|\psi\rangle\right|$ *generally valid*?

Definition of error & disturbance

- Error is defined as the root-mean-square (rms):

$$\varepsilon(A) \equiv \langle\psi\otimes\xi|\left[U^{\dagger}(I\otimes M)U - A\otimes I\right]^2|\psi\otimes\xi\rangle^{1/2}$$

- Disturbance is defined in the same manner:

$$\eta(B) \equiv \langle\psi\otimes\xi|\left[U^{\dagger}(B\otimes I)U - B\otimes I\right]^2|\psi\otimes\xi\rangle^{1/2}$$

Definition: $(\mathcal{K}, \xi, U, M)$ is a measuring process

$\mathcal{K}$: a Hilbert space,
ξ: a unit vector in $\mathcal{K}$,
U: a unitary operator on $\mathcal{H}\otimes\mathcal{K}$,
M: a self-adjoint operator on $\mathcal{K}$.

M. Ozawa,
Ann. Phys **311**, 350 (2004).

Universally valid uncertainty relation by Ozawa 1

Joint measurement of A and B in state ψ:

- $M^{out} = A^{in} + N(A)$
- $B^{out} = B^{in} + D(A)$

We obtain the following commutation relation for noise and disturbance operator with $[M^{out}, B^{out}]=0$

$$[N(A), D(B)] + [N(A), B^{in}] + [A^{in}, D(B)] \geq -[A^{in}, B^{in}]$$

applying the triangular inequality

$$|\langle[N(A), D(B)]\rangle| + |\langle[N(A), B^{in}]\rangle + \langle[A^{in}, D(B)]\rangle| \geq |Tr([A, B]\rho)|$$

M. Ozawa, Phys. Rev. A **67**, 042105 (2003).

Universally valid uncertainty relation by Ozawa 2

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2}|\langle\psi|[A, B]|\psi\rangle|$$

$$\begin{cases} \varepsilon: \text{error of the first measurmen} (A) \\ \eta: \text{disturbance on the second measurement } (B) \\ \sigma: \text{standard deviations} \end{cases}$$

First term: error of the first measuremt, disturbance on the second measurement

second and third terms: crosstalks between spreads of wavefunctions and error/disturbance

M. Ozawa, Phys. Rev. A **67**, 042105 (2003).

量子物理学の根幹原理：新旧の不確定性関係

旧 不確定性関係（ハイゼンベク）

$$\Delta q \ \Delta p \ \ge \ \frac{h}{4\pi}$$

<教科書にのっている>

正確に位置を測ろうとすると
運動量(速度)がかわってしまう！
⇩
位置か運動量(速度)の
どちらか一方しか正確に測れない！
⇩
2つの測定精度のトレードオフ関係

新 不確定性関係（小澤）

$$\Delta q \ \Delta p + \Delta q \ \sigma_p + \sigma_q \ \Delta p \ge \frac{h}{4\pi}$$

h:プランク定数
Δq：位置の誤差
Δp：運動量の擾乱
σ_p, σ_q：標準偏差

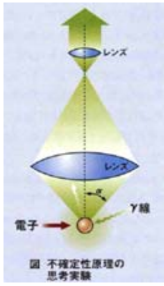


図 不確定性原理の思考実験

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

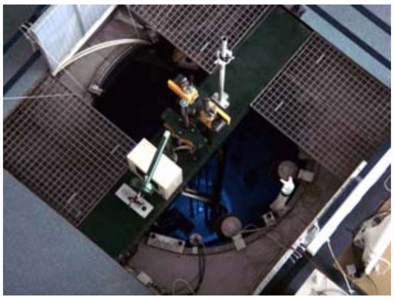
ウィーン工科大学
Atominsitut

ウィーンの物理学者１

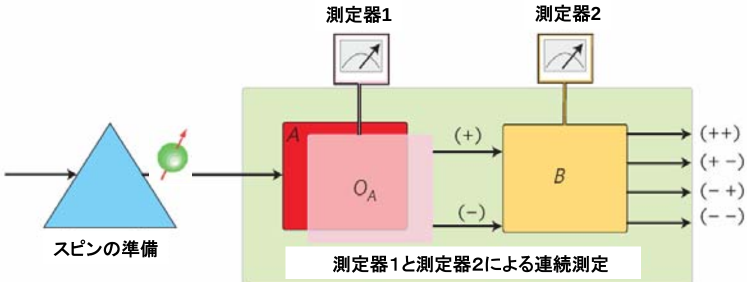
ウィーンの物理学者２

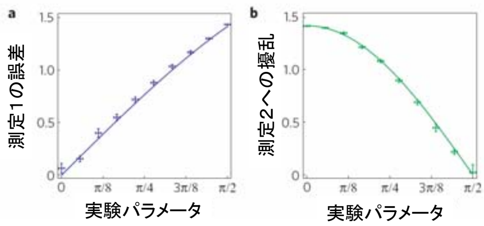
TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

トリガ型
研究用原子炉



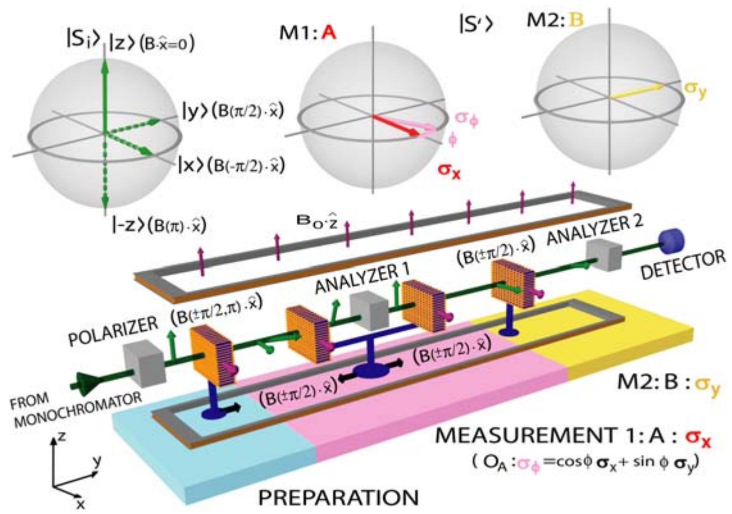
実験の概念図：連続測定による誤差と擾乱の確定



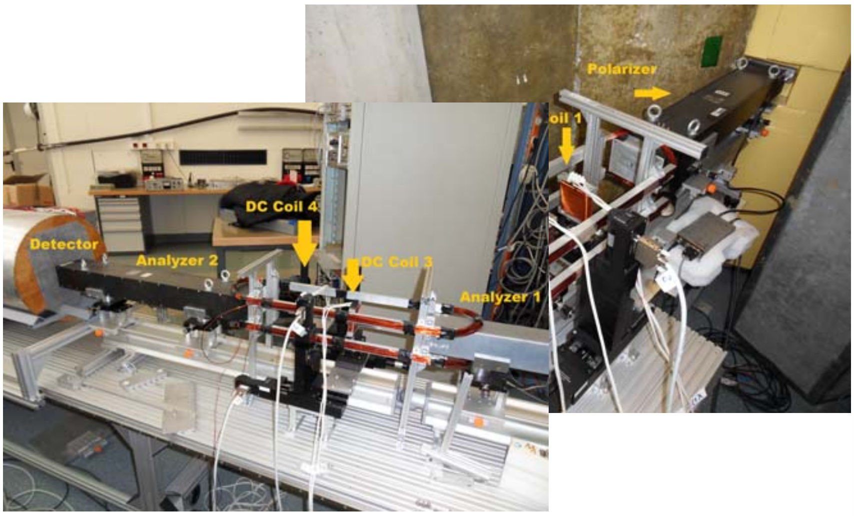


2つの測定精度の
トレードオフ関係

Experimental setup



Experimental setup

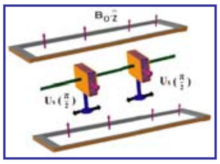


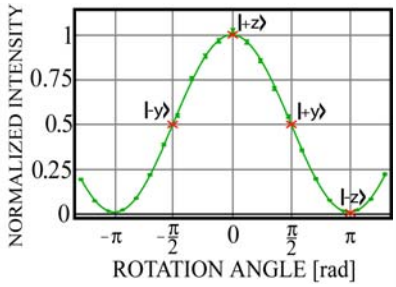
Adjustment

Unitary transformation: $U=e^{i\alpha\sigma/2}$

Determination of $U(\pm\pi)$
- Scanning of the current in x-direction

Determination of $U(\pm\pi/2)$
- Position scanning





Contrast of each DC coil adjustement: ~98%

Contrast of the whole system: ~96%

Experimental determination

where I_+ and I_- represent the positive and negative projections

$$\sigma^2(\hat{\sigma}_j) = \langle \hat{\sigma}_j{}^2 \rangle - \langle \hat{\sigma}_j \rangle^2 = 1 - (\langle \sigma_+ \rangle - \langle \sigma_- \rangle)^2 = 1 - \left(\frac{I_+ - I_-}{I_+ + I_-} \right)^2$$

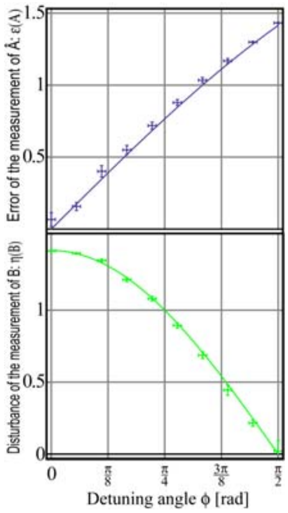
Projection operator: $\vec{P} := \langle \psi | \vec{\sigma} | \psi \rangle$

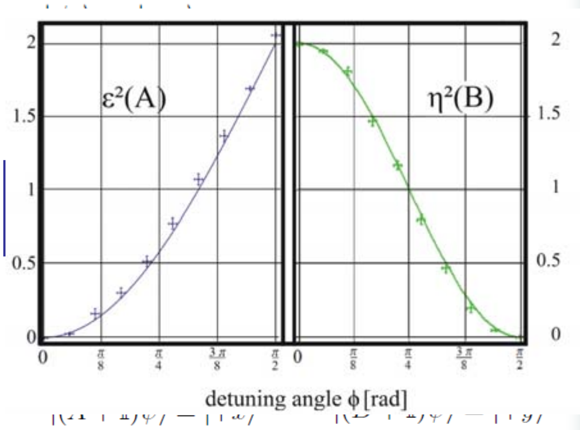
Error of A (projective measurment):

$$\epsilon(A)^2 = \underbrace{\langle \psi | A^2 | \psi \rangle}_{1} + \underbrace{\langle \psi | O_A^2 | \psi \rangle}_{1} + \langle \psi | O_A | \psi \rangle + \langle A\psi | O_A | A\psi \rangle - \langle (A + \mathbb{I})\psi | O_A | (A + \mathbb{I})\psi \rangle$$

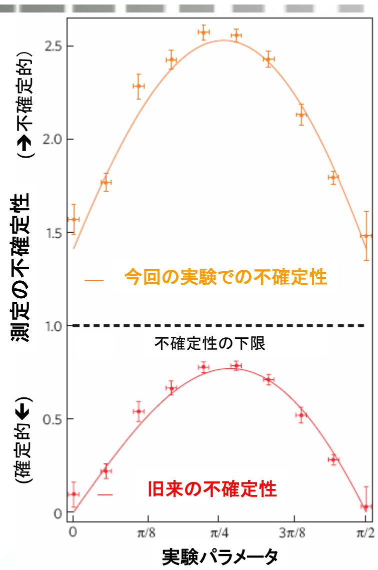
$$\langle \psi | O_A | \psi \rangle = \frac{(I_{++} + I_{+-}) - (I_{-+} + I_{--})}{\sum I}$$

Results: error-disturbance trade-off





実験結果：新旧不確定性関係の比較対照



今回の実験での不確定性は常に下限よりも大きい。

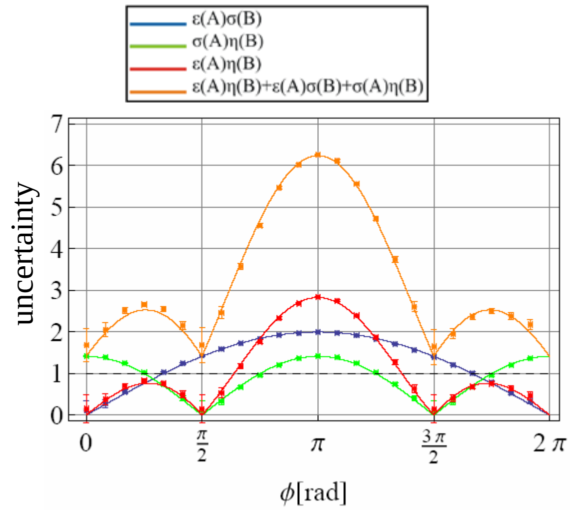
➔小澤の不等式（成立）

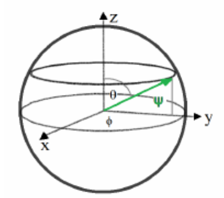
$$\Delta q\ \Delta p + \Delta q\ \sigma_p + \sigma_q\ \Delta p \geq \frac{h}{4\pi}$$

旧来の不確定性は下限より小さい

➔ハイゼンベルクの不等式（不成立） $\Delta q\ \Delta p \ngeq \dfrac{h}{4\pi}$

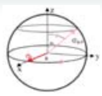
Results 1: incident spin-state (|s>=|θ,ϕ>)



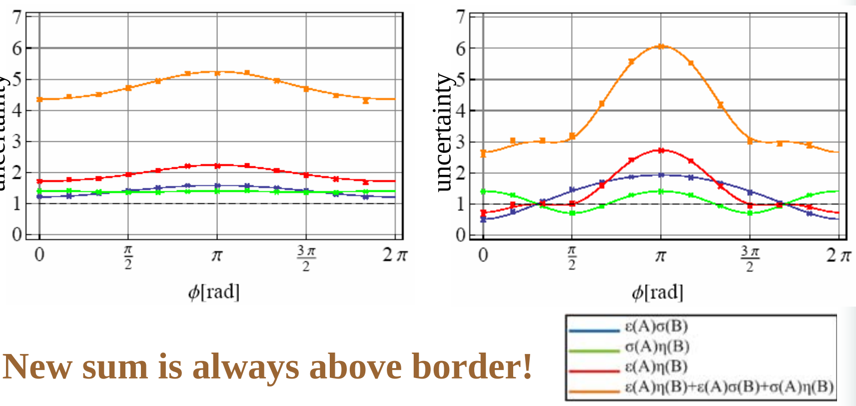


$$|s\rangle = |\theta = 0, \phi = 0\rangle$$

Results 3: polar angle of O_A [θ(O_A)]

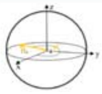


$\theta(O_A) = \pi/12$ $\theta(O_A) = \pi/3$

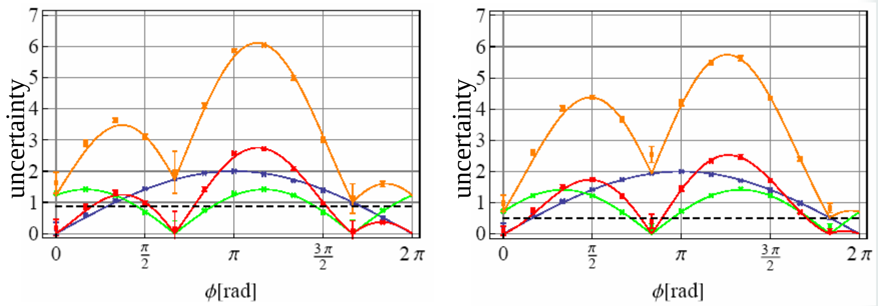


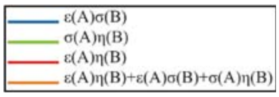
New sum is always above border!

Results 4: azimuthal angle of B [ϕ(B)]



$\phi(B) = 2\pi/3$ $\phi(B) = 5\pi/6$



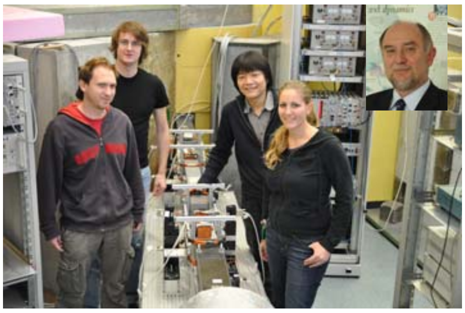


Asymmetry appears!
Touch the border!

Concluding remarks: universally valid uncertain-relation

Universally valid uncertainty-relation by Ozawa is experimentally tested!

- **Neutron's spin measurement revealed the new error-disturbance uncertainty relation.**
- **Error & disturbance are determined from data.**
  Projective measurements are exploited.
- **New sum is always above the limit!**
  Heisenberg product is often below the limit!



Neutron experiments in future

□ Interferometry & Polarimetry @ILL, ATI

- Entanglements, phase space tomography, decoherence
- Berry/topological phases, phases in a moving frame
- *new aspects of wave-particle duality*

□ Ultra Cold Neutrons (UCNs) @ILL

- Berry/topological phases
- Phase space transformer
- Measurements of nuclear-nuclear interactions in extreme situations

□ Spin Echo Spectrometer @ISIS

- Gravitationally induced phase, Compton frequency(?)
- Topological phases

□ Phase Contrast Imagings @ILL (in cooperation with RIKEN)

- Applications of neutron's matter-wave interference to imagings

Fundamental/applied studies with neutron's matter-wave!